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Left-Right Gauge Model in Nonassociative Geometry

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Abstract

We reformulate the left-right gauge model of Pati-Mohapatra using nonassociative geometry approach. At the tree level we obtain the mass relations $M_W = \frac{1}{2}m_t$, $M_H = \frac{3}{2}m_t$ and the mixing angles are $\sin^2\theta_W = \frac{3}{8}$ and $\sin^2\theta_s = \frac{3}{5}$, which are identical to the ones obtained in $SO(10)$ GUT .

1 Introduction

One of the greatest achievement of noncommutative geometry (abbreviated **NCG** hereafter) is its geometrization of the standard model ([1],[2],[3]). NCG provides a framework where the Higgs boson H may be introduced on the same level as $W^{\pm}$ and Z bosons. In this approach we introduce additional discrete dimensions to the four-dimensional space-time. If the gauge bosons are associated to the continuous directions, the Higgs boson results from gauging the discrete directions. However one of the shortcomings of NCG is its non accessibility to grand unified theories GUT ([4],[5]). The other formulation of NCG due to Coquereaux [6] does not suffer from this problem ([7]-[10]). There are some variant of Connes' theory ([11],[12]) where we use an auxiliary Hilbert space to fit GUT.

R. Wulkenhaar has recently succeeded in formulating another type of geometry which shares a lot of common points with NCG à la Connes [13]. The theory was baptized "nonassociative geometry" **NAG**. The main difference with the two theories is that NAG is based on unitary Lie algebra instead of unital associative $\star$-algebra in the case of NCG. Its application to several physical models has been successful ([14]-[17]).

We try using this approach to reformulate the left-right model LRM ([18],[19]). One of the features of NAG (and NCG) is its geometric explanation of the spontaneous breakdown of gauge symmetry. Our aim is to investigate whether NAG could encompass also the parity violation.

In section 2 we present the main elements to formulate gauge theories using NAG. In section 3 we derive the bosonic action of left-right model and finally we discuss our results.

2 Some elements of Nonassociative Geometry

NAG is based on the L-cycle $(\mathfrak{g},\mathcal{H}, D, \pi, \Gamma)$ [13]

$$\mathfrak{g} = C^{\infty}(X) \otimes \mathfrak{a} \tag{1}$$

where $C^{\infty}(X)$ is the algebra of smooth functions on the manifold X and $\mathfrak{a}$ is the matrix Lie algebra. $\mathcal{H}$ is the Hilbert space

$$\mathcal{H} = L^2(X, S) \otimes \mathbb{C}^F \tag{2}$$

where $L^2(X,S)$ is the Hilbert space of spinors.

D is the total Dirac operator given by:

$$D = \mathbf{D} \otimes \mathbf{1}_F + \gamma^5 \otimes \mathcal{M} \tag{3}$$

where $\mathbf{D}$ is the Dirac operator associated with the continuous algebra $C^\infty(X)$ and $\mathcal{M}$ is associated to the discrete algebra $\mathfrak{a}$.

The representation of $\mathfrak{g}$ on $\mathcal{H}$ is given by

$$\pi = \mathbf{1} \otimes \hat{\pi}, \tag{4}$$

where $\hat{\pi}$ is the representation of $\mathfrak{a}$ on $\mathbb{C}^F$.

Finally, the graded operator Γ, acting on $\mathcal{H}$, is given by:

$$\Gamma = \gamma^5 \otimes \hat{\Gamma}, \tag{5}$$

where $\hat{\Gamma}$ is the grading operator acting on $\mathbb{C}^F$.

The space $\hat{\Omega}^1\mathfrak{a}$ is generated by the elements of the type

$$\omega^1 = \sum_z [a^z, ...[a^1, da^0]...] \qquad a^i \in \mathfrak{a}. \tag{6}$$

The representation $\hat{\pi}$ acts on the space $\hat{\Omega}^1\mathfrak{a}$ as:

$$\hat{\pi} : \hat{\Omega}^1\mathfrak{a} \longrightarrow \mathbb{M}_F(\mathbb{C})$$

$$\tau^1 = \hat{\pi}(\omega^1) := \sum_z [\hat{\pi}(a^z), ...[\hat{\pi}(a^1), [-i\mathcal{M}, \hat{\pi}(a^0)]]...] \tag{7}$$

We define also the mapping:

$$\hat{\sigma} : \hat{\Omega}^1\mathfrak{a} \longrightarrow \mathbb{M}_F(\mathbb{C})$$

$$\hat{\sigma}(\omega^1) := \sum_z [\hat{\pi}(a^z), ...[\hat{\pi}(a^1), [\mathcal{M}^2, \hat{\pi}(a^0)]]...] \tag{8}$$

For n$\geqslant 2$ we have

$$\pi(J^{k+1}\mathfrak{g}) = \left\{ \sigma(\omega^k); \omega^k \in \hat{\Omega}^k\mathfrak{g} \cap \ker(\pi) \right\}. \tag{9}$$

The main difference between NCG and NAG is that in NAG the connection ρ and the curvature θ are not, in general, elements of $\Omega^1\mathfrak{g}$ and $\Omega^2\mathfrak{g}$. To construct connection and curvature we need the spaces $\mathbf{r}^0\mathfrak{a}$, $\mathbf{r}^1\mathfrak{a} \subset M_F(\mathbb{C})$ defined by the conditions:

$$\begin{aligned}
\mathfrak{r}^0(\mathfrak{a}) &= \hat{\Gamma}\mathfrak{r}^0(\mathfrak{a})\hat{\Gamma} \\
\left[\mathfrak{r}^0(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\right] &\subset \hat{\pi}(\mathfrak{a}) \\
\left[\mathfrak{r}^0(\mathfrak{a}), \hat{\pi}(\Omega^1\mathfrak{a})\right] &\subset \hat{\pi}(\Omega^1\mathfrak{a}) \\
\left\{\mathfrak{r}^0(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\right\} &\subset \{\hat{\pi}(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\} + \hat{\pi}(\Omega^2\mathfrak{a}) \\
\left[\mathfrak{r}^0(\mathfrak{a}), \hat{\pi}(\Omega^1\mathfrak{a})\right] &\subset \{\hat{\pi}(\mathfrak{a}), \hat{\pi}(\Omega^1\mathfrak{a})\} + \hat{\pi}(\Omega^3\mathfrak{a})
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
\mathfrak{r}^1(\mathfrak{a}) &= -\hat{\Gamma}\mathfrak{r}^1(\mathfrak{a})\hat{\Gamma} \\
\left[\mathfrak{r}^1(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\right] &\subset \hat{\pi}(\Omega^1\mathfrak{a}) \\
\left\{\mathfrak{r}^1(\mathfrak{a}), \hat{\pi}(\Omega^1\mathfrak{a})\right\} &\subset \hat{\pi}(\Omega^2\mathfrak{a}) + \{\hat{\pi}(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\}
\end{aligned} \tag{11}$$

where $\hat{\Gamma}$ is the grading operator.

The connection is given by:

$$\rho = \sum_{\alpha} (c^1_\alpha \otimes m^0_\alpha + c^0_\alpha \gamma^5 \otimes m^1_\alpha) \tag{12}$$

where

$$c^1_\alpha \in \Lambda^1, c^0_\alpha \in \Lambda^0 \tag{13}$$

Λ^k is the space of k-forms represented by gamma matrices, and

$$\begin{aligned}
m^0_\alpha &\in \mathfrak{r}^0(\mathfrak{a}) \\
m^1_\alpha &\in \mathfrak{r}^1(\mathfrak{a})
\end{aligned} \tag{14}$$

To construct the curvature θ we need the spaces $\mathbf{j}^0\mathfrak{a}$, $\mathbf{j}^1\mathfrak{a}$ and $\mathbf{j}^2\mathfrak{a}$ defined as:

$$\mathbf{j}^0\mathfrak{a} = \mathbf{c}^0\mathfrak{a} \tag{15}$$

where

$$\begin{aligned}
\mathbf{c}^0\mathfrak{a} &= \mathbf{c}^0\mathfrak{a}\ \hat{\Gamma}\ \mathbf{c}^0\mathfrak{a} \\
\mathbf{c}^0\mathfrak{a}\ .\hat{\pi}(a) &= 0 \\
\mathbf{c}^0\mathfrak{a}\ .\hat{\pi}(\Omega^1 a) &= 0
\end{aligned} \tag{16}$$

and

$$\mathbf{j}^1\mathfrak{a} = \mathbf{c}^1\mathfrak{a} \tag{17}$$

where

$$\begin{aligned} \mathbf{c}^1\mathfrak{a} &= -\mathbf{c}^1\mathfrak{a}\ \hat{\Gamma}\ \mathbf{c}^1\mathfrak{a} \\ \mathbf{c}^1\mathfrak{a}\ .\hat{\pi}(a) &= 0 \\ \mathbf{c}^1\mathfrak{a}\ .\hat{\pi}(\Omega^1 a) &= 0 \end{aligned} \tag{18}$$

and

$$\mathbf{j}^2\mathfrak{a} = \mathbf{c}^2\mathfrak{a} + \hat{\pi}(\mathfrak{J}^2\mathfrak{a}) + \{\hat{\pi}(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\} \tag{19}$$

with

$$\mathbf{c}^2\mathfrak{a}\ = \mathbf{c}^2\mathfrak{a}\ \hat{\Gamma}\ \mathbf{c}^2\mathfrak{a}$$

$$\begin{aligned} \left[\mathbf{c}^2\mathfrak{a}\ , \hat{\pi}(\mathfrak{a})\right] &= 0 \\ \left[\mathbf{c}^2\mathfrak{a}\ , \hat{\pi}(\Omega^1\mathfrak{a})\right] &= 0 \end{aligned} \tag{20}$$

The curvature θ is given by:

$$\theta = \mathbf{d}\rho + \rho^2 - i\{\gamma^5 \otimes \mathcal{M}, \rho\} + \hat{\sigma}_{\mathfrak{g}}(\rho)\gamma^5 + \mathbb{J}^2\mathfrak{g} \tag{21}$$

where

$$\mathbb{J}^2\mathfrak{g} = (\Lambda^2 \otimes j^0\mathfrak{a}) \oplus (\Lambda^1\gamma^5 \otimes j^1\mathfrak{a}) \oplus (\Lambda^0 \otimes j^2\mathfrak{a}) \tag{22}$$

We have to choose the representative of the curvature θ , $\varepsilon(\theta)$, orthogonal to $\mathbb{J}^2\mathfrak{g}$,given by:

$$\varepsilon(\theta) = \mathbf{d}\rho + \rho^2 - i\{\gamma^5 \otimes \mathcal{M}, \rho\} + \hat{\sigma}_{\mathfrak{g}}(\rho)\gamma^5 + j \tag{23}$$

where

$$\int_X dxtr(\varepsilon(\theta)j') = 0 \ \ \forall j' \in \mathbb{J}^2\mathfrak{g} \tag{24}$$

The bosonic and the fermionic actions are given by:

$$S_B = \frac{1}{N_F g_0^2}\int_X dxtr(\varepsilon(\theta)^2) \tag{25}$$

and

$$S_F = \int_X \Psi^*(D + i\rho)\Psi \tag{26}$$

where g_0 is a coupling constant.

3 The construction

The discrete L-cycle $(\mathfrak{a},\mathcal{H},\mathcal{M})$ consists of the Lie algebra given by:

$$\mathfrak{a} = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1) \tag{27}$$

$$\ni \{a_3, a_2, a_2', a_1\}$$

The Hilbert space $\mathcal{H}$ is $\mathbb{C}^{48}$ labelled by the elements

$$(\mathbf{u}_L, \mathbf{d}_L, \mathbf{u}_R, \mathbf{d}_R, \nu_L, e_L, \nu_R, e_R)^T$$

where $\mathbf{u}_L, \mathbf{d}_L, \mathbf{u}_R, \mathbf{d}_R \in \mathbb{C}^3 \otimes \mathbb{C}^3$ and $\nu_L, e_L, \nu_R, e_R \in \mathbb{C}^3$.

The Lie algebra $\mathfrak{a}$ acts on $\mathcal{H}$ via the representation:

$$\hat{\pi}((a_1, a_2, a_2', a_3)) = \begin{bmatrix} \hat{\pi}_q((a_1, a_2, a_2', a_3)) & 0 \\ 0 & \hat{\pi}_l((a_1, a_2, a_2', a_3)) \end{bmatrix} \tag{28}$$

where q and l label, respectively, the quarkionic and the leptonic sectors, with the representation of the quark sector given by:

$$\hat{\pi}_q((a_1, a_2, a_2', a_3)) = \begin{bmatrix} \hat{\pi}_q^1((a_1, a_2, a_2', a_3)) & 0 \\ 0 & \hat{\pi}_q^2((a_1, a_2, a_2', a_3)) \end{bmatrix} \tag{29}$$

where

$$\begin{aligned} \hat{\pi}_q^1((a_1, a_2, a_2', a_3)) &= if_0 diag(\alpha \mathbf{1}_3 \otimes \mathbf{1}_3, \beta \mathbf{1}_3 \otimes \mathbf{1}_3) + \\ & \begin{bmatrix} (a_3 + if_3^1 \mathbf{1}_3) \otimes \mathbf{1}_3 & i(f_1^1 - if_2^1)\mathbf{1}_3 \otimes \mathbf{1}_3 \\ i(f_1^1 + if_2^1)\mathbf{1}_3 \otimes \mathbf{1}_3 & (a_3 - if_3^1 \mathbf{1}_3) \otimes \mathbf{1}_3 \end{bmatrix} \end{aligned} \tag{30}$$

and

$$\begin{aligned} \hat{\pi}_q^2((a_1, a_2, a_2', a_3)) &= if_0 diag(\gamma \mathbf{1}_3 \otimes \mathbf{1}_3, \delta \mathbf{1}_3 \otimes \mathbf{1}_3) + \\ & \begin{bmatrix} (a_3 + if_3^2 \mathbf{1}_3) \otimes \mathbf{1}_3 & i(f_1^2 - if_2^2)\mathbf{1}_3 \otimes \mathbf{1}_3 \\ i(f_1^2 + if_2^2)\mathbf{1}_3 \otimes \mathbf{1}_3 & (a_3 - if_3^2 \mathbf{1}_3) \otimes \mathbf{1}_3 \end{bmatrix} \end{aligned} \tag{31}$$

and the representation of the leptonic sector:

$$\hat{\pi}_l((a_1, a_2, a_2', a_3)) = \begin{bmatrix} \hat{\pi}_l^1((a_1, a_2, a_2', a_3)) & 0 \\ 0 & \hat{\pi}_l^2((a_1, a_2, a_2', a_3)) \end{bmatrix} \tag{32}$$

where

$$\hat{\pi}_l^1((a_1, a_2, a_2', a_3)) = if_0 diag(\upsilon \mathbf{1}_3, \varepsilon \mathbf{1}_3) + \left[\begin{array}{ll} if_3^1 \otimes \mathbf{1}_3 & i(f_1^1 - if_2^1) \otimes \mathbf{1}_3 \\ i(f_1^1 + if_2^1) \otimes \mathbf{1}_3 & -if_3^1 \otimes \mathbf{1}_3 \end{array}\right] \tag{33}$$

and

$$\hat{\pi}_l^2((a_1, a_2, a_2', a_3)) = if_0 diag(\zeta \mathbf{1}_3, \eta \mathbf{1}_3) + \left[\begin{array}{ll} if_3^2 \otimes \mathbf{1}_3 & i(f_1^2 - if_2^2) \otimes \mathbf{1}_3 \\ i(f_1^2 + if_2^2) \otimes \mathbf{1}_3 & -if_3^2 \otimes \mathbf{1}_3 \end{array}\right] \tag{34}$$

where we have used the fact that elements of $su(2)$ algebra have the representation:

$$a_2 = \left[\begin{array}{ll} if_3 & i(f_1 - if_2) \\ i(f_1 + if_2) & -if_3 \end{array}\right]. \tag{35}$$

The Dirac operator is given by:

$$\mathcal{M} = \left[\begin{array}{ll} \mathcal{M}_q & 0 \\ 0 & \mathcal{M}_l \end{array}\right] \tag{36}$$

with

$$\mathcal{M}_q = \left[\begin{array}{cccc} 0 & 0 & \mathbf{1}_3 \otimes M_u & \mathbf{0} \\ 0 & 0 & 0 & \mathbf{1}_3 \otimes M_d \\ \mathbf{1}_3 \otimes M_u^* & 0 & 0 & 0 \\ 0 & \mathbf{1}_3 \otimes M_d^* & 0 & 0 \end{array}\right] \tag{37}$$

and

$$\mathcal{M}_l = \left[\begin{array}{cccc} 0 & 0 & M_\nu & \mathbf{0} \\ 0 & 0 & 0 & M_e \\ M_\nu^* & 0 & 0 & 0 \\ 0 & M_e^* & 0 & 0 \end{array}\right] \tag{38}$$

where $M_u, M_d, M_\nu, M_e \in M_3(\mathbb{C})$ are the mass matrices of the fermions.

Remark that we have chosen the coefficients associated with u(1) algebra arbitrary, we will see when calculating the fermionic action that in fact they corresponds to the hypercharges.

The grading operator is given by:

$$\hat{\Gamma} = diag(-\mathbf{1}_3 \otimes \mathbf{1}_3, -\mathbf{1}_3 \otimes \mathbf{1}_3, \mathbf{1}_3 \otimes \mathbf{1}_3, \mathbf{1}_3 \otimes \mathbf{1}_3, -\mathbf{1}_3, -\mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3) \tag{39}$$

The space $\hat{\pi}(\Omega^1 a)$ is generated by elements of the type

$$\tau^1 = \sum_{\alpha,z} \left[\hat{\pi}(a^z_\alpha), ... \left[\hat{\pi}(a^1_\alpha), \left[-i\mathcal{M}, \hat{\pi}(a^0_\alpha)\right]\right] ...\right] \tag{40}$$

where $a^i_\alpha = (a^i_1, a^i_2, a^{i\prime}_2, a^i_3)$.

Simple calculation gives the following decomposition:

$$\tau^1 = \begin{bmatrix} \tau^1_q & 0 \\ 0 & \tau^1_l \end{bmatrix} \tag{41}$$

where for the quark sector we have

$$\tau^1_q = \begin{bmatrix} 0 & \tau^1_{1q} \\ \tau^1_{2q} & 0 \end{bmatrix} \tag{42}$$

with

$$\tau^1_{1q} = i \begin{bmatrix} \bar{b}_2 1_3 \otimes M_d + \bar{c}_2 1_3 \otimes M_u & b_1 1_3 \otimes M_d + c_1 1_3 \otimes M_u \\ -\bar{b}_1 1_3 \otimes M_u - \bar{c}_1 1_3 \otimes M_d & b_2 1_3 \otimes M_u + c_2 1_3 \otimes M_d \end{bmatrix} \tag{43}$$

and

$$\tau^1_{2q} = i \begin{bmatrix} b_2 1_3 \otimes M^*_d + c_2 1_3 \otimes M^*_u & -b_1 1_3 \otimes M^*_u - c_1 1_3 \otimes M^*_d \\ \bar{b}_1 1_3 \otimes M^*_d + \bar{c}_1 1_3 \otimes M^*_u & \bar{b}_2 1_3 \otimes M^*_u + \bar{c}_2 1_3 \otimes M^*_d \end{bmatrix}. \tag{44}$$

For the leptonic sector we have

$$\tau^1_l = \begin{bmatrix} 0 & \tau^1_{1l} \\ \tau^1_{2l} & 0 \end{bmatrix} \tag{45}$$

where

$$\tau^1_{1l} = i \begin{bmatrix} \bar{b}_2 \otimes M_e + \bar{c}_2 \otimes M_\nu & b1 \otimes M_e + c_1 \otimes M_\nu \\ -\bar{b}_1 \otimes M_\nu - \bar{c}_1 \otimes M_e & b_2 \otimes M_\nu + c_2 \otimes M_e \end{bmatrix} \tag{46}$$

and

$$\tau^1_{2l} = i \begin{bmatrix} b_2 \otimes M^*_\nu + c_2 \otimes M^*_e & -b_1 \otimes M^*_\nu - c_1 \otimes M^*_e \\ \bar{b}_1 \otimes M^*_e + \bar{c}_1 \otimes M^*_\nu & \bar{b}_2 \otimes M^*_e + \bar{c}_2 \otimes M^*_\nu \end{bmatrix}. \tag{47}$$

We can show that the form $\tau^1$is antihermetian provided that:

$$\alpha + \beta = \gamma + \delta \tag{48}$$

and

$$\upsilon + \varepsilon = \zeta + \eta. \tag{49}$$

Similarly, straightforward calculations give:

$$\hat{\pi}(\Omega^2\mathfrak{a}) \ni \tau^2 = \{\tau^1, \tau^1\} = \begin{bmatrix} \tau_q^2 & 0 \\ 0 & \tau_l^2 \end{bmatrix} \tag{50}$$

with

$$\tau_l^2 = \begin{bmatrix} \tau_{11}^{l2} & \tau_{12}^{l2} & 0 & 0 \\ \tau_{21}^{l2} & \tau_{22}^{l2} & 0 & 0 \\ 0 & 0 & \tau_{33}^{l2} & \tau_{34}^{l2} \\ 0 & 0 & \tau_{43}^{l2} & \tau_{44}^{l2} \end{bmatrix} \tag{51}$$

where the diagonal elements are:

$$\begin{aligned} -\tau_{11}^{l2} &= 2(|b_1|^2 + |c_2|^2) \otimes M_e M_e^* + 2(|c_1|^2 + |b_2|^2) \otimes M_\nu M_\nu^* \\ &\quad +2(b_1\bar{c}_1 + \bar{c}_2 b_2) \otimes M_e M_v^* + 2(\bar{b}_1 c_1 + c_2\bar{b}_2) \otimes M_\nu M_e^* \\ -\tau_{22}^{l2} &= 2(|c_1|^2 + |b_2|^2) \otimes M_e M_e^* + 2(|b_1|^2 + |c_2|^2) \otimes M_\nu M_\nu^* \\ &\quad +2(b_1\bar{c}_1 + \bar{c}_2 b_2) \otimes M_e M_v^* + 2(\bar{b}_1 c_1 + c_2\bar{b}_2) \otimes M_\nu M_e^* \\ -\tau_{33}^{l2} &= 2(|c_2|^2 + |c_1|^2) \otimes M_e M_e^* + 2(|b_1|^2 + |b_2|^2) \otimes M_\nu M_\nu^* \\ &\quad +2(b_1\bar{c}_1 + \bar{c}_2 b_2) \otimes M_e M_v^* + 2(\bar{b}_1 c_1 + c_2\bar{b}_2) \otimes M_\nu M_e^* \\ -\tau_{44}^{l2} &= 2(|b_1|^2 + |b_2|^2) \otimes M_e M_e^* + 2(|c_1|^2 + |c_2|^2) \otimes M_\nu M_\nu^* \\ &\quad +2(b_1\bar{c}_1 + \bar{c}_2 b_2) \otimes M_e M_v^* + 2(\bar{b}_1 c_1 + c_2\bar{b}_2) \otimes M_\nu M_e^* \end{aligned} \tag{52}$$

and the off-diagonal ones:

$$\begin{aligned} -\tau_{12}^{l2} &= 2(\bar{c}_2 c_1 - b_1\bar{b}_2) \otimes M_{\nu e} \\ -\tau_{21}^{l2} &= 2(c_2\bar{c}_1 - \bar{b}_1 b_2) \otimes M_{\nu e} \\ -\tau_{34}^{l2} &= 2(c_1 b_2 - b_1 c_2) \otimes M_{\nu e} \\ -\tau_{43}^{l2} &= 2(\bar{c}_1\bar{b}_2 - \bar{b}_1\bar{c}_2) \otimes M_{\nu e} \end{aligned} \tag{53}$$

with

$$\begin{aligned} M_{\nu e} &= M_\nu M_\nu^* - M_e M_e^* \\ M_{\{\nu e\}} &= M_\nu M_\nu^* + M_e M_e^* \end{aligned} \tag{54}$$

and similarly

$$\tau_q^2 = \begin{bmatrix} \tau_{11}^{q2} & \tau_{12}^{q2} & 0 & 0 \\ \tau_{21}^{q2} & \tau_{22}^{q2} & 0 & 0 \\ 0 & 0 & \tau_{33}^{q2} & \tau_{34}^{q2} \\ 0 & 0 & \tau_{43}^{q2} & \tau_{44}^{q2} \end{bmatrix} \tag{55}$$

with

$$
\begin{aligned}
-\tau_{11}^{q2} &= 2(|b_1|^2+|c_2|^2)\mathbf{1}_3\otimes M_dM_d^* + 2(|c_1|^2+|b_2|^2)\mathbf{1}_3\otimes M_uM_u^* \\
&\quad +2(b_1\bar{c}_1+\bar{c}_2b_2)\mathbf{1}_3\otimes M_dM_u^* + 2(\bar{b}_1c_1+c_2\bar{b}_2)\mathbf{1}_3\otimes M_uM_d^* \\
-\tau_{22}^{l2} &= 2(|c_1|^2+|b_2|^2)\mathbf{1}_3\otimes M_dM_d^* + 2(|b_1|^2+|c_2|^2)\mathbf{1}_3\otimes M_uM_u^* \\
&\quad +2(b_1\bar{c}_1+\bar{c}_2b_2)\mathbf{1}_3\otimes M_dM_u^* + 2(\bar{b}_1c_1+c_2\bar{b}_2)\mathbf{1}_3\otimes M_uM_d^* \\
-\tau_{33}^{l2} &= 2(|c_2|^2+|c_1|^2)\mathbf{1}_3\otimes M_dM_d^* + 2(|b_1|^2+|b_2|^2)\mathbf{1}_3\otimes M_uM_u^* \\
&\quad +2(b_1\bar{c}_1+\bar{c}_2b_2)\mathbf{1}_3\otimes M_dM_u^* + 2(\bar{b}_1c_1+c_2\bar{b}_2)\mathbf{1}_3\otimes M_uM_d^* \\
-\tau_{44}^{l2} &= 2(|b_1|^2+|b_2|^2)\mathbf{1}_3\otimes M_dM_d^* + 2(|c_1|^2+|c_2|^2)\mathbf{1}_3\otimes M_uM_u^* \\
&\quad +2(b_1\bar{c}_1+\bar{c}_2b_2)\mathbf{1}_3\otimes M_dM_u^* + 2(\bar{b}_1c_1+c_2\bar{b}_2)\mathbf{1}_3\otimes M_uM_d^*
\end{aligned}
\tag{56}
$$

and

$$
\begin{aligned}
-\tau_{12}^{q2} &= 2(\bar{c}_2c_1-b_1\bar{b}_2)\mathbf{1}_3\otimes M_{ud} \\
-\tau_{21}^{q2} &= 2(c_2\bar{c}_1-\bar{b}_1b_2)\mathbf{1}_3\otimes M_{ud} \\
-\tau_{34}^{q2} &= 2(c_1b_2-b_1c_2)\mathbf{1}_3\otimes M_{ud} \\
-\tau_{43}^{q2} &= 2(\bar{c}_1\bar{b}_2-\bar{b}_1\bar{c}_2)\mathbf{1}_3\otimes M_{\nu ud}
\end{aligned}
\tag{57}
$$

where

$$
\begin{aligned}
M_{ud} &= M_uM_u^* - M_dM_d^* \\
M_{\{ud\}} &= M_uM_u^* + M_dM_d^*.
\end{aligned}
\tag{58}
$$

We can show that:

$$
\hat{\sigma}(\omega^1) = \left[\begin{array}{ll} \hat{\sigma}_q(\omega^1) & 0 \\ 0 & \hat{\sigma}_l(\omega^1) \end{array}\right]
\tag{59}
$$

where

$$
\hat{\sigma}_q(\omega^1) = \left[\begin{array}{ll} \hat{\sigma}_{1q} & 0 \\ 0 & \hat{\sigma}_{2q} \end{array}\right]
\tag{60}
$$

with

$$
\hat{\sigma}_{1q} = i\left[\begin{array}{ll} if_3^1 1_3\otimes M_{ud} & i(f_1^1-if_2^1)1_3\otimes M_{ud} \\ i(f_1^1+if_2^1)1_3\otimes M_{ud} & -if_3^1 1_3\otimes M_{ud} \end{array}\right]
\tag{61}
$$

and

$$
\hat{\sigma}_{2q} = i\left[\begin{array}{ll} if_3^2 1_3\otimes M_{ud} & i(f_1^2-if_2^2)1_3\otimes M_{ud} \\ i(f_1^2+if_2^2)1_3\otimes M_{ud} & -if_3^2 1_3\otimes M_{ud} \end{array}\right].
\tag{62}
$$

For the leptonic sector we have

$$\hat{\sigma}_l(\omega^1) = \begin{bmatrix} \hat{\sigma}_{1l} & 0 \\ 0 & \hat{\sigma}_{2l} \end{bmatrix} \tag{63}$$

with

$$\hat{\sigma}_{1l} = i \begin{bmatrix} if_3^1 \otimes M_{\nu e} & i(f_1^1 - if_2^1) \otimes M_{\nu e} \\ i(f_1^1 + if_2^1) \otimes M_{\nu e} & -if_3^1 \otimes M_{\nu e} \end{bmatrix} \tag{64}$$

and

$$\hat{\sigma}_{2l} = i \begin{bmatrix} if_3^2 \otimes M_{\nu e} & i(f_1^2 - if_2^2) \otimes M_{\nu e} \\ i(f_1^2 + if_2^2) \otimes M_{\nu e} & -if_3^2 \otimes M_{\nu e} \end{bmatrix}. \tag{65}$$

After trivial calculation we can show that

$$\begin{aligned} \tau^2 \;&=\; diag(K' \otimes 1_3, K' \otimes 1_3, K' \otimes 1_3, K' \otimes 1_3, \\ &\quad K, K, K, K) \bmod(\hat{\sigma}(\Omega^1 a)) \end{aligned} \tag{66}$$

with

$$K = \hat{\alpha}\ M_{\{\nu e\}} + \hat{\beta}\ M_e M_v^* + \hat{\gamma}\ M_\nu M_e^* \tag{67}$$

and

$$K' = \hat{\alpha}\ M_{\{ud\}} + \hat{\beta}\ M_d M_u^* + \hat{\gamma}\ M_u M_d^* \tag{68}$$

where

$$\begin{aligned} \hat{\alpha} &= |b_1|^2 + |b_2|^2 + |c_1|^2 + |c_2|^2 \\ \hat{\beta} &= b_1\bar{c}_1 + b_2\bar{c}_2 \\ \hat{\gamma} &= \bar{\hat{\beta}} = \bar{b}_1 c_1 + \bar{b}_2 c_2. \end{aligned} \tag{69}$$

Using the conditions given by eqs (10,11), we can show that:

$$\begin{aligned} \mathfrak{r}^0(\mathfrak{a}) &= \hat{\pi}(\mathfrak{a}) \\ \mathfrak{r}^1(\mathfrak{a}) &= \hat{\pi}(\Omega^1\mathfrak{a}) \end{aligned} \tag{70}$$

Let us adopt the following parametrization

$$\sum_\alpha c_\alpha^1 \otimes a_{1\alpha} = \sum_\alpha c_\alpha^1 \otimes if_{0\alpha} = i\tilde{\mathbf{A}} \in \Lambda^1 \otimes u(1) = i\frac{g'}{2}\gamma^\mu C_\mu \tag{71}$$

$$\sum_\alpha c^1_\alpha \otimes a_{3\alpha} = \hat{\mathbf{A}} \in \Lambda^1 \otimes su(3) \tag{72}$$

$$\begin{aligned}
\sum_\alpha c^1_\alpha \otimes a_{2\alpha} &= A \in \Lambda^1 \otimes su(2) \\
&= \left[\begin{array}{ll} \sum\limits_\alpha c^1_\alpha \otimes i f_{3\alpha} & \sum\limits_\alpha c^1_\alpha \otimes i(f_{1\alpha} - i f_{2\alpha}) \\ \sum\limits_\alpha c^1_\alpha \otimes i(f_{1\alpha} + i f_{2\alpha}) & \sum\limits_\alpha c^1_\alpha \otimes (-i f_{3\alpha}) \end{array} \right] \\
&= \left[\begin{array}{ll} iA_3 & i(A_1 - iA_2) \\ i(A_1 + iA_2) & -iA_3 \end{array} \right]
\end{aligned} \tag{73}$$

and

$$\begin{aligned}
\Phi_i &= -\sum_\alpha c^0_\alpha \otimes b_{i\alpha} \in \Lambda^0 \otimes \mathbb{C} \\
\Xi_i &= -\sum_\alpha c^0_\alpha \otimes c_{i\alpha} \in \Lambda^0 \otimes \mathbb{C}
\end{aligned} \tag{74}$$

The connection ρ is given by

$$\rho = \left[\begin{array}{ll} \rho_q & 0 \\ 0 & \rho_l \end{array} \right] \tag{75}$$

where

$$\rho_l = \left[\begin{array}{ll} \rho_l^{A_1} & \rho_l^{H_1} \\ \rho_l^{H2} & \rho_l^{A_2} \end{array} \right] \tag{76}$$

with

$$\begin{aligned}
\rho_l^{A_1} &= \left[\begin{array}{ll} i(\upsilon\tilde{A} + A_3) \otimes 1_3 & i(A_1 - iA_2) \otimes 1_3 \\ i(A_1 + iA_2) \otimes 1_3 & i(\varepsilon\tilde{A} - A_3) \otimes 1_3 \end{array} \right] \\
\rho_l^{A_2} &= \left[\begin{array}{ll} i(\zeta\tilde{A} + A'_3) \otimes 1_3 & i(A'_1 - iA'_2) \otimes 1_3 \\ i(A'_1 + iA'_2) \otimes 1_3 & i(\eta\tilde{A} - A'_3) \otimes 1_3 \end{array} \right] \\
\rho_l^{H_1} &= \left[\begin{array}{ll} -i\gamma^5\bar{\Phi}_2 \otimes M_e - i\gamma^5\bar{\Xi}_2 \otimes M_\nu & -i\gamma^5\Phi_1 \otimes M_e - i\gamma^5\Xi_1 \otimes M_\nu \\ i\gamma^5\bar{\Phi}_1 1_3 \otimes M_\nu + i\gamma^5\bar{\Xi}_1 \otimes M_e & -i\gamma^5\Phi_2 \otimes M_\nu - i\gamma^5\Xi_2 \otimes M_e \end{array} \right] \\
\rho_l^{H2} &= \left[\begin{array}{ll} -i\gamma^5\Phi_2 \otimes M^*_e - i\gamma^5\Xi_2 \otimes M^*_\nu & i\gamma^5\Phi_1 \otimes M^*_\nu + i\gamma^5\Xi_1 \otimes M^*_e \\ -i\gamma^5\bar{\Phi}_1 \otimes M^*_e - i\gamma^5\bar{\Xi}_1 \otimes M^*_\nu & -i\gamma^5\bar{\Phi}_2 \otimes M^*_\nu - i\gamma^5\bar{\Xi}_2 \otimes M^*_e \end{array} \right]
\end{aligned} \tag{77}$$

and

$$\rho_q = \begin{bmatrix} \rho_q^{A_1} & \rho_q^{H_1} \\ \rho_q^{H2} & \rho_q^{A_2} \end{bmatrix} \tag{78}$$

with

$$\rho_q^{A_1} = \begin{bmatrix} (\hat{A} + i(\alpha\tilde{A} + A_3)1_3) \otimes 1_3 & i(A_1 - iA_2)1_3 \otimes 1_3 \\ i(A_1 + iA_2)1_3 \otimes 1_3 & (\hat{A} + i(\beta\tilde{A} - A_3)1_3) \otimes 1_3 \end{bmatrix}$$

$$\rho_q^{A_2} = \begin{bmatrix} (\hat{A} + i(\gamma\tilde{A} + A_3')1_3) \otimes 1_3 & i(A_1' - iA_2')1_3 \otimes 1_3 \\ i(A_1' + iA_2')1_3 \otimes 1_3 & (\hat{A} + i(\delta\tilde{A} - A_3')1_3) \otimes 1_3 \end{bmatrix} \tag{79}$$

$$\rho_q^{H_1} = \begin{bmatrix} -i\gamma^5\bar{\Phi}_2 1_3 \otimes M_d - i\gamma^5\bar{\Xi}_2 1_3 \otimes M_u & -i\gamma^5\Phi_1 1_3 \otimes M_d - i\gamma^5\Xi_1 1_3 \otimes M_u \\ i\gamma^5\bar{\Phi}_1 1_3 \otimes M_u + i\gamma^5\bar{\Xi}_1 1_3 \otimes M_d & -i\gamma^5\Phi_2 1_3 \otimes M_u - i\gamma^5\Xi_2 1_3 \otimes M_d \end{bmatrix}$$

$$\rho_q^{H_2} = \begin{bmatrix} -i\gamma^5\Phi_2 1_3 \otimes M_d^* - i\gamma^5\Xi_2 1_3 \otimes M_e^* & i\gamma^5\Phi_1 1_3 \otimes M_u^* + i\gamma^5\Xi_1 1_3 \otimes M_d^* \\ -i\gamma^5\bar{\Phi}_1 1_3 \otimes M_d^* - i\gamma^5\bar{\Xi}_1 1_3 \otimes M_e^* & -i\gamma^5\bar{\Phi}_2 1_3 \otimes M_u^* - i\gamma^5\bar{\Xi}_2 1_3 \otimes M_d^* \end{bmatrix}$$

The interaction fermions-bosons is given by

$$\mathfrak{L}_{F-B} = i\Psi^*\rho\Psi = \mathfrak{L}^{lB} + \mathfrak{L}^{qB} + \mathfrak{L}^{lH} + \mathfrak{L}^{qH}, \tag{80}$$

where $\mathfrak{L}^{lB}, \mathfrak{L}^{qB}; \mathfrak{L}^{lH}$, $\mathfrak{L}^{qH}$ are ,respectively, the Lagrangians of interaction leptons-bosons, quarks-bosons, leptons-Higgs and quark-Higgs, and

$$\Psi = (\mathbf{u}_L, \mathbf{d}_L, \mathbf{u}_R, \mathbf{d}_R, \nu_L, e_L, \nu_R, e_R)^T. \tag{81}$$

Using the parametrization

$$\tilde{\mathbf{A}} = \mathbf{i}\frac{g'}{2}\gamma^\mu C_\mu \tag{82}$$

$$\mathbf{A} = i\frac{g_L}{2}\sum_{a=1}^{3} W_\mu^a\gamma^\mu \otimes \sigma^a \tag{83}$$

$$\mathbf{A}' = i\frac{g_R}{2}\sum_{a=1}^{3} W_\mu'^a\gamma^\mu \otimes \sigma^a \tag{84}$$

and

$$\hat{\mathbf{A}} = i\frac{g_s}{2}\sum_{a=1}^{3} G_\mu^a\gamma^\mu \otimes \lambda^a \tag{85}$$

where $g', g_L,\ g_R,\ g_s$ are respectively the coupling constants of $u(1)_{L-B}$, $su(2)_L$, $su(2)_R$ and $su(3)$ algebras.

Let [20]

$$
\begin{aligned}
B_\mu &= \cos(\theta_W)A_\mu - \sin(\theta_W)Z_\mu \\
W^3_\mu &= \sin(\theta_W)A_\mu + \cos(\theta_W)Z_\mu
\end{aligned}
\tag{86}
$$

and

$$
\begin{aligned}
C_\mu &= \cos(\theta_S)B_\mu - \sin(\theta_S)Z'_\mu \\
W'^3_\mu &= \sin(\theta_S)B_\mu + \cos(\theta_S)Z'_\mu,
\end{aligned}
\tag{87}
$$

where the field B_μ is associated to the group $U(1)_Y$, and θ_W and θ_S are the mixing angles. Once performing a Wick rotation and by identification with the fermionic action provided by the classical model we obtain [22]:

$$
\varepsilon = \eta = \upsilon = \zeta = -1 \tag{88}
$$

which are the leptons hypercharges in the left-right model [21].

The same treatment, applied to the quarkionic sector, gives

$$
\alpha = \beta = \gamma = \delta = \frac{1}{3} \tag{89}
$$

which are the quark hypercharges.

Using the conditions given by eqs. (16, 18 and 20) we can show that

$$
\mathbf{j}^0\mathfrak{a} = 0 \tag{90}
$$

$$
\mathbf{j}^1\mathfrak{a} = 0 \tag{91}
$$

and

$$
\mathbf{j}^2\mathfrak{a} = diag(\mathbb{R}\mathbf{I}_{36}, \mathbb{R}\mathbf{I}_{12}) + \hat{\pi}(\mathfrak{J}^2\mathfrak{a}) + \{\hat{\pi}(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\}\,. \tag{92}
$$

Let the anticommutator

$$
\hat{\psi}(\mathfrak{a}) = \{\hat{\pi}(\mathfrak{a}), \hat{\pi}(\mathfrak{a})\} = diag(\{\hat{\pi}_q(\mathfrak{a}), \hat{\pi}_q(\mathfrak{a})\}, \{\hat{\pi}_l(\mathfrak{a}), \hat{\pi}_l(\mathfrak{a})\}) \tag{93}
$$

where

$$\{\hat{\pi}_q(a), \hat{\pi}_q(a)\} \ni A_q + \Delta_q \tag{94}$$

with

$$A_q = \begin{bmatrix} A_q^1 & 0 \\ 0 & A_q^2 \end{bmatrix} \otimes I_3 \tag{95}$$

where

$$A_q^1 = i \begin{bmatrix} \lambda_1^0 a_3 + i\theta_1^0 I_3 & (\varpi^1 - i\varpi^2)a_3 + i(\hat{\lambda}^1 - i\hat{\lambda}^2)I_3 \\ (\varpi^1 + i\varpi^2)a_3 + i(\hat{\lambda}^1 + i\hat{\lambda}^2)I_3 & \lambda_2^0 a_3 - i\theta_1^0 I_3 \end{bmatrix} \tag{96}$$

$$A_q^2 = i \begin{bmatrix} \lambda_3^0 a_3 + i\theta_2^0 I_3 & (\varpi^3 - i\varpi^4)a_3 + i(\hat{\lambda}^3 - i\hat{\lambda}^4)I_3 \\ (\varpi^3 + i\varpi^4)a_3 + i(\hat{\lambda}^3 + i\hat{\lambda}^4)I_3 & \lambda_4^0 a_3 - i\theta_2^0 I_3 \end{bmatrix} \tag{97}$$

and

$$\Delta_q = diag(\lambda_1 \mathbf{I}_3, \lambda_1 \mathbf{I}_3, \lambda_2 \mathbf{I}_3, \lambda_2 \mathbf{I}_3) \otimes \mathbf{I}_3 \tag{98}$$

with the condition

$$\lambda_1^0 + \lambda_2^0 = \lambda_3^0 + \lambda_4^0 \tag{99}$$

and

$$\{\hat{\pi}_l(a), \hat{\pi}_l(a)\} \ni A_l + \Delta_l \tag{100}$$

with

$$A_l = i \begin{bmatrix} i\kappa_1^0 I_3 & i(\check{\lambda}^1 - i\check{\lambda}^2)I_3 & 0 & 0 \\ i(\check{\lambda}^1 + i\check{\lambda}^2)I_3 & -i\kappa_1^0 I_3 & 0 & 0 \\ 0 & 0 & i\kappa_2^0 I_3 & i(\check{\lambda}^3 - i\check{\lambda}^4)I_3 \\ 0 & 0 & i(\check{\lambda}^3 + i\check{\lambda}^4)I_3 & -i\kappa_2^0 I_3 \end{bmatrix} \tag{101}$$

and

$$\Delta_l = diag(\chi \mathbf{I}_3, \chi \mathbf{I}_3, \chi \mathbf{I}_3, \chi \mathbf{I}_3) \tag{102}$$

with $\lambda_i, \lambda_i^0, \check{\lambda}^i, \hat{\lambda}^i, \theta_j^0, \kappa_j^0, \varpi^i, \chi$ $(i = 1..4,\ j = 1..2) \in \mathbb{R}$.

Trivial calculations give [22]

$$\mathbf{j}^2 a = \hat{\pi}(\mathcal{J}^2\mathfrak{a}) \oplus diag(A_q, A_l) \oplus diag(K_q, K_l) \tag{103}$$

where

$$\begin{aligned} K_q &= diag(\varphi_1\mathbf{I}_3, \varphi_1\mathbf{I}_3, \varphi_2\mathbf{I}_3, \varphi_2\mathbf{I}_3) \otimes \mathbf{I}_3 \\ K_l &= diag(\varphi_3\mathbf{I}_3, \varphi_3\mathbf{I}_3, \varphi_3\mathbf{I}_3, \varphi_3\mathbf{I}_3) \end{aligned} \tag{104}$$

with φ_1, φ_2 et $\varphi_3 \in \mathbb{R}$.

The representative $\varepsilon(\{\tau^1, \tau^1\})$ of $\{\tau^1, \tau^1\} + \mathbf{j}^2 a$ orthogonal to $\mathbf{j}^2 a$ is given by:

$$\begin{aligned} \varepsilon(\left\{\tau^1, \tau^1\right\}) &= diag(\tilde{K}' \otimes 1_3, \tilde{K}' \otimes 1_3, \tilde{K}' \otimes 1_3, \tilde{K}' \otimes 1_3, \\ &\quad \tilde{K}, \tilde{K}, \tilde{K}, \tilde{K}) \operatorname{mod}(\hat{\sigma}(\Omega^1 a)) \end{aligned} \tag{105}$$

with

$$\begin{aligned} \tilde{K} &= \hat{\alpha}\ \tilde{M}_{\{\nu e\}} + \hat{\beta}\ \ \overset{\smile}{M_e M_v^*} + \hat{\gamma}\ \ \overset{\smile}{M_\nu M_e^*} \\ \tilde{K}' &= \hat{\alpha}\ \tilde{M}_{\{ud\}} + \hat{\beta}\ \ \overset{\smile}{M_d M_u^*} + \hat{\gamma}\ \ \overset{\smile}{M_u M_d^*} \end{aligned} \tag{106}$$

where

$$\tilde{M}_{\{\nu e\}} = M_\nu M_\nu^* + M_e M_e^* - \frac{1}{3} tr(M_\nu M_\nu^* + M_e M_e^*)\mathbf{1}_3,$$

$$\overset{\smile}{M_e M_v^*} = M_e M_v^* - \frac{1}{3} tr(M_e M_v^*)\mathbf{1}_3,$$

$$\overset{\smile}{M_\nu M_e^*} = M_\nu M_e^* - \frac{1}{3} tr(M_\nu M_e^*)\mathbf{1}_3, \tag{107}$$

$$\tilde{M}_{\{ud\}} = \ = M_u M_u^* + M_d M_d^* - \frac{1}{3} tr(M_u M_u^* + M_d M_d^*)\mathbf{1}_{3,}$$

$$\overset{\smile}{M_d M_u^*} = M_d M_u^* - \frac{1}{3} tr(M_d M_u^*)\mathbf{1}_{3,}$$

and

$$\overset{\smile}{M_u M_d^*} = M_u M_d^* - \frac{1}{3} tr(M_u M_d^*)\mathbf{1}_3.$$

Remark also that

$$\begin{aligned}\mathcal{M}^2 &= \frac{1}{2} diag(1_3 \otimes \tilde{M}_{\{ud\}}, 1_3 \otimes \tilde{M}_{\{ud\}}, 1_3 \otimes \tilde{M}_{\{ud\}}, 1_3 \otimes \tilde{M}_{\{ud\}}, \\ &\quad \tilde{M}_{\{\nu e\}}, \tilde{M}_{\{\nu e\}}, \tilde{M}_{\{\nu e\}}, \tilde{M}_{\{\nu e\}}) \bmod(\hat{\sigma}(\Omega^1 a)) \end{aligned} \tag{108}$$

The bosonic action is given by

$$S_B = \int_X (\mathfrak{L}_0 + \mathfrak{L}_1 + \mathfrak{L}_2) dx \tag{109}$$

where

$$\mathfrak{L}_0 = \frac{1}{96 g_0^2} \{ tr(12 \tilde{M}^2_{\{ud\}} + 4 \tilde{M}^2_{\{\nu e\}}) \times (|\Phi_1|^2 + |\Phi_2|^2 + |\Xi_1|^2 + |\Xi_2 + 1|^2 - 1)^2$$

$$+ tr(12(\breve{M}_d M_u^*)^2 + 4(\breve{M}_e M_\nu^*)^2) \times (\Phi_1 \ \bar{\Xi}_1 + \Phi_2 (\bar{\Xi}_2 + 1))$$

$$+ tr(12(\breve{M}_u M_d^*)^2 + 4(\breve{M}_\nu M_e^*)^2) \times (\bar{\Phi}_1 \ \Xi_1 + \bar{\Phi}_2 \ (\Xi_2 + 1)) \} \tag{110}$$

is the Higgs Lagrangian,
and

$$\begin{aligned}\mathfrak{L}_2 &= \frac{1}{4g_0^2} tr_c(\mathbf{d}\ \hat{\mathbf{A}} + \frac{1}{2} \left\{ \hat{\mathbf{A}}, \hat{\mathbf{A}} \right\})^2 + \frac{1}{4g_0^2} tr_c((\mathbf{d}\ \mathbf{A} + \frac{1}{2} \{\mathbf{A}, \mathbf{A}\})^2) \\ &\quad + \frac{1}{4g_0^2} tr_c(\mathbf{d}\ \mathbf{A}' + \frac{1}{2} \{\mathbf{A}', \mathbf{A}'\})^2 + \frac{1}{3g_0^2} tr_c((\mathbf{d}\ \tilde{\mathbf{A}}\ )^2)) \end{aligned} \tag{111}$$

is the Yang-Mills Lagrangian.

Since $\mathfrak{L}_1$, the Lagrangian of interaction Higgs-bosons, is quite long it is not displayed here in its totality [22]. The parts of $\mathfrak{L}_1$ which give rise to the masses of W_R and W_L bosons are respectively:

$$\mathcal{L}_1^1 = \frac{1}{8g_0^2} \mathbf{m}^2 tr_c \left| i(A_1 - iA_2)(\Xi_2 + 1) \right|^2 \tag{112}$$

and

$$\mathcal{L}_1^2 = \frac{1}{8g_0^2} \mathbf{m}^2 tr_c \left| i(A_1' - iA_2')(\Xi_2 + 1) \right|^2, \tag{113}$$

where

$$\mathbf{m}^2 = tr(\frac{1}{3}M_e M_e^* + \frac{1}{3}M_\nu M_\nu^* + M_u M_u^* + M_d M_d^*). \tag{114}$$

Let us make the following rescaling

$$\Xi_2 = \frac{\bar{g}\phi_2}{\mathbf{m}}, \tag{115}$$

where $\bar{g}$ is a coupling constant which is specified later.

Since

$$\begin{aligned} i(A_1 - iA_2) &= i\frac{g_L}{\sqrt{2}} W^\dagger_{\mu L}\gamma^\mu \\ i(A_1' - iA_2') &= i\frac{g_L}{\sqrt{2}} W^\dagger_{\mu R}\gamma^\mu \end{aligned} \tag{116}$$

and

$$tr_c(\gamma^\mu\gamma^\nu) = 4\delta^{\mu\nu} \tag{117}$$

then

$$\mathcal{L}_1^1 = \frac{1}{4}(\frac{\bar{g}g_L}{g_0})^2 W^\dagger_{\mu L} W^\mu_L \phi_2^2 + \frac{1}{2}(\frac{g_L}{g_0})^2 \mathbf{m}\bar{g} W^\dagger_{\mu L} W^\mu_L \phi_2 + \frac{1}{4}(\frac{g_L}{g_0})^2 \mathbf{m}^2 W^\dagger_{\mu L} W^\mu_L \tag{118}$$

The last term gives:

$$M_{W_L} = \frac{1}{2}(\frac{g_L}{g_0})\mathbf{m} \tag{119}$$

We can show also that:

$$M_{W_R} = \frac{1}{2}(\frac{g_R}{g_0})\mathbf{m} \tag{120}$$

Hence

$$\frac{M_{W_L}}{M_{W_R}} = \frac{g_L}{g_R} \tag{121}$$

In the case of left-right symmetric model

$$g_L = g_{R,} \tag{122}$$

we have

$$M_{W_L} = M_{W_R}. \tag{123}$$

and the left-handed and the right-handed gauge bosons acquire the same masses, leading to the conclusion that parity violation does not occur for such models ([23],[24]). The same conclusion has been obtained by [25] when discussing the $U(2)_L \times U(2)_R$ model in the framework of noncommutative geometry à la Connes.

For left-right symmetric model the nonassociative geometry approach does not accommodate parity violation to be contrasted to Coquereaux approach [21].

Remark that if we neglect the mass of fermions in respect of the mass of the top quark m_t we obtain

$$M_{W_L} = \frac{1}{2}(\frac{g_L}{g_0})m_t \tag{124}$$

and

$$M_{W_R} = \frac{1}{2}(\frac{g_R}{g_0})m_t. \tag{125}$$

The masses Z et Z' bosons come from

$$\mathcal{L}_1^3 = \frac{1}{8g_0^2}\mathbf{m}^2 tr_c \left| i(A_0 - iA_3)(\Xi_2 + 1) \right|^2 \tag{126}$$

and

$$\mathcal{L}_1^4 = \frac{1}{8g_0^2}\mathbf{m}^2 tr_c \left| i(A_0 - iA_3')(\Xi_2 + 1) \right|^2. \tag{127}$$

The calculations give

$$M_Z = \frac{\mathbf{m}}{2}(\frac{g_L}{g_0})\frac{1}{\cos(\theta_W)}. \tag{128}$$

Since

$$M_{W_L} = \frac{1}{2}(\frac{g_L}{g_0})\mathbf{m} \tag{129}$$

then

$$M_Z = \frac{M_{W_L}}{\cos(\theta_W)}, \tag{130}$$

which is the same relation as in the standard model.

Y. Okumura [21] has found the same relation when applying noncommutative geometry à la Coquereaux to left-right gauge model.

Similarly, we obtain

$$M_{z'} = \frac{\mathbf{m}}{2}(\frac{g'}{g_0})\sin(\theta_s). \tag{131}$$

From eq (110) the free Lagrangian associated with the field Ξ_2 is given by:

$$\begin{aligned} \mathfrak{L}^{\Xi_2}_{free} &= \frac{1}{8g_0^2}\Upsilon^2(|\Xi_2+1|^2-1)^2 \\ &= \frac{1}{2}\left(\frac{\bar{g}}{g_0}\right)^2\left(\frac{\Upsilon}{\mathbf{m}}\right)^2\phi_2^2 + \text{higher order terms} \end{aligned} \tag{132}$$

where

$$\Upsilon^2 = tr(\tilde{M}^2_{\{ud\}} + \frac{1}{3}\tilde{M}^2_{\{\nu e\}}). \tag{133}$$

Hence

$$m_{\phi_2} = (\frac{\bar{g}}{g_0})\left(\frac{\Upsilon}{\mathbf{m}}\right). \tag{134}$$

If we take $\bar{g} = g_0$, then

$$m_{\phi_2} = \sqrt{\frac{tr(\tilde{M}^2_{\{ud\}} + \frac{1}{3}\tilde{M}^2_{\{\nu e\}})}{tr(M_uM_u^* + M_dM_d^* + \frac{1}{3}M_eM_e^* + \frac{1}{3}M_\nu M_\nu^*)}}. \tag{135}$$

Neglecting the mass of fermions in respect to the mass of the top quark gives:

$$m_{\phi_2} \approx \sqrt{\frac{tr(\tilde{M}^2_{\{ud\}})}{tr(M_uM_u^*)}} = \frac{3}{2}m_t \tag{136}$$

which is the same relation found by Wulkenhaar for the neutrinos mass matrix $M_\nu \neq 0$ [14].

Let us now turn to the mixing angles, eq (111) looks like its counterpart in classical model provided that:

$$g_s = g_L = g_R = g_0 \tag{137}$$

and

$$g' = \sqrt{\frac{3}{2}}g_0 \tag{138}$$

which give:

$$\sin^2(\theta_W) = \frac{3}{8} \tag{139}$$

and

$$\sin^2(\theta_S) = \frac{3}{5} \tag{140}$$

which are the same angles given by $SO(10)$ GUT. However du to eq.(138) the grand unification for coupling constants is not achieved.

4 Conclusion

We reformulate the left-right model in the framework of nonassociative geometry. We have found that the left-right symmetric model does not exhibit the parity violation, however NAG gives the mixing angles predicted by SO(10) GUT and at the tree level we have the mass relations

$$M_{W_L} = \frac{1}{2}(\frac{g_L}{g_0})m_t \tag{141}$$

$$M_{W_R} = \frac{1}{2}(\frac{g_R}{g_0})m_t \tag{142}$$

$$M_Z = \frac{M_{W_L}}{\cos(\theta_W)} \tag{143}$$

$$M_{Z'} = \frac{g' \sin(\theta_S)}{g_R} M_{W_R} \tag{144}$$

and

$$M_H = \frac{3}{2} m_t \tag{145}$$

where m_t is the top mass.

The unification of the coupling constants is not achieved using this reformulation.

The main result of this work is that nonassociative geometry, like noncommutative geometry, does not provide a geometrical explanation of the parity violation.

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